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illustration from one's everyday experience often wonderfully clears up hazy notions will perhaps justify the following allegory, which has been successful in bringing forth that first, wide-eyed, understanding "oh!" in several cases.

A long—indeed apparently endlessly long—line of people is seen to be standing waiting to buy tickets at a window. One wonders if some far-famed magician is to give an exhibition of extracting square roots from apparently empty silk hats. Our hero, Mr. Kueedee, being the proprietor of a rival attraction, wishes to persuade this line of people to stop waiting, and come to his show. He looks at the people, and seeing a friend, Mr. Kayplusone, he goes over to him.

"Kayplusone," he says earnestly, "I feel that you are a typical average sort of a chap. I come to you to see if I can learn the attitude which all these people take towards the proposition of coming at once to my show."

"I have indeed been thinking that matter over," replies Kayplusone, "and I will agree to come if my friend Kay ahead of me here in line will also go. And I can save you a lot of questions by telling you that mine is a typical attitude. Kay says that he, in turn, will go if the man ahead of him will."

Kueedee ponders for but a moment, and then with sudden inspiration goes to the end of the line, and is seen to be in serious conversation with the man next to the ticket window. Suddenly this man, whose name happens to be One, grabs Kueedee by the hand and calls out, "I'll do it." He speaks hurriedly to the man behind him, and starts off. This man, likewise, speaks to the man behind him, and starts off also. So that Mr. Kueedee goes, calm in the confidence that even the infinite capacity of his house is going to be taxed, and himself puts out the sign,

STANDING ROOM ONLY.

Q. E. D.

III. THE CURVE OF CONCENTRATION FOR A LIQUID MIXTURE.

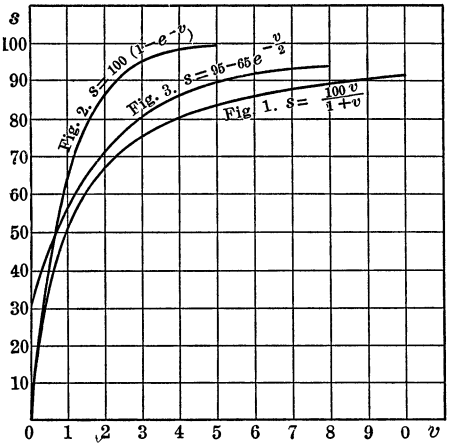
By C. A. NOBLE, University of California.

When strong alcohol is added to weak alcohol the concentration of the mixture may be representated by the arc of a hyperbola. If, for example, to a volume v_0, of percentage-strength s_0, is added a volume v, of percentage-strength s_1, the strength of the mixture is given by

$$s = \frac{s_0 v_0 + s_1 v}{v_0 + v}.$$

Figure 1 shows the curve of concentration for the case where pure alcohol is added to a unit volume of pure water. Then $s_0 = 0$, $s_1 = 100$, $v_0 = 1$, and the equation of the curve is

$$s = \frac{100v}{1 + v}.$$



One can obtain more rapidly a desired volume of required concentration if one keeps the volume of the mixture constant, by drawing off from the stirred mixture a volume equal to that of the strong alcohol which has been added. The smaller the steps taken, in this process of adding a small quantity, stirring the mixture, and then pouring off an equal quantity, the more rapidly does the mixture increase in strength. The following discussion is for the case where the process goes on by infinitesimal steps, *i. e.*, continuously.

With the notation mentioned above, let the volume $\Delta v = v/n$ be added to v_0, the mixture stirred, and then the volume Δv of the mixture poured off. The strength of the remaining mixture (volume v_0) is

$$\frac{s_0 v_0 + s_1 \Delta v}{v_0 + \Delta v}.$$

Abbreviate this to k/v_1. Again add Δv of the strong alcohol, stir the mixture, and pour from it Δv. The strength of the mixture is now

$$\frac{s_1 v_1 \Delta v + k v_0}{v_1^2}.$$

After another repetition of this process, the mixture has the strength

$$\frac{s_1 \Delta v (v_1^2 + v_0 v_1) + k v_0^2}{v_1^3}.$$

After $n - 1$ such repetitions, the mixture has the strength

$$s_n = \frac{s_1 \Delta v (v_1^{n-1} + v_0 v_1^{n-2} + \cdots + v_0^{n-2} v_1) + k v_0^{n-1}}{v_1^n}$$

$$= \frac{s_1 \Delta v}{v_1} \cdot \frac{1 - \left(\frac{v_0}{v_1}\right)^{n-1}}{1 - \left(\frac{v_0}{v_1}\right)} + \frac{k}{v_1}\left(\frac{v_0}{v_1}\right)^{n-1}.$$

If the process is to be continuous, n must become infinite, and the limit s of s_n will be the ultimate concentration. Now,

$$\underset{n=\infty}{\text{limit}} \left(\frac{v_0}{v_1}\right)^{n-1} = \underset{n=\infty}{\text{limit}} \left[\frac{1}{1 + \frac{v/v_0}{n}}\right]^{n-1} = e^{-v/v_0},$$

and

$$\lim_{n=\infty}\frac{1-\left(\frac{v_0}{v_1}\right)^{n-1}}{1-\frac{v_0}{v_1}}\cdot\Delta v=\frac{v(1-e^{-v/v_0})}{\lim\limits_{n=\infty} n\left[1-\frac{v_0}{v_0+\frac{v}{n}}\right]}=v_0(1-e^{-v/v_0}).$$

Moreover,

$$\lim_{n=\infty}\frac{s_1}{v_1}=\frac{s_1}{v_0},$$

and

$$\lim_{n=\infty}\frac{k}{v_1}=s_0.$$

Inserting these values in the above expression for s_n, one has

$$s=\lim_{n=\infty} s_n=s_1-(s_1-s_0)e^{-v/v_0}.$$

This curve has a steeper slope than the hyperbola mentioned above, that is, this method of continuous dilution is the more economical of the two. It is actually employed in biological work at the University of California by Professor Long, who has devised appropriate apparatus, and at whose suggestion the equation of the curve which should represent the process was deduced.

Figure 2 is the curve which represents the pouring of pure alcohol into one unit volume of pure water. $s_1=100$, $s_0=0$, $v_0=1$, and the equation is

$$s=100(1-e^{-v}).$$

Figure 3 represents the pouring of 95 per cent. alcohol into two unit volumes of 30 per cent. alcohol. $s_1=95$, $s_0=30$, $v_0=2$, and the equation is

$$s=95-65e^{-v/2}.$$

The differential equation corresponding to this process is simple, and may be deduced as follows:

Let s be the strength of the mixture, the volume of which is maintained at v_0 in the way above indicated. When the small volume Δv of strength s_1 is added, the strength of the mixture changes to

$$\frac{sv_0+s_1\Delta v}{v_0+\Delta v}.$$

The increment which this brings to s is

$$\Delta s=\frac{sv_0+s_1\Delta v}{v_0+\Delta v}-s=\frac{(s_1-s)\Delta v}{v_0+\Delta v}.$$

The difference-quotient becomes

$$\frac{\Delta s}{\Delta v}=\frac{s_1-s}{v_0+\Delta v}.$$

Hence, the rate of change of the strength of the mixture with respect to the volume of liquid added, is given by

$$\frac{ds}{dv} = \frac{s_1 - s}{v_0} \quad \text{or} \quad \frac{ds}{s_1 - s} = \frac{dv}{v_0}.$$

The value of v, the volume necessary to raise the strength from s_0 to s, is given by

$$\frac{v}{v_0} = \int_{s_0}^{s} \frac{ds}{s_1 - s} = -\log \frac{s_1 - s}{s_1 - s_0},$$

i. e., by

$$s = s_1 - (s_1 - s_0)e^{-v/v_0}$$

as before.

February, 1919.

RECENT PUBLICATIONS.

REVIEWS.

VEBLEN AND YOUNG'S PROJECTIVE GEOMETRY.

Projective Geometry. By O. VEBLEN and J. W. YOUNG. Boston: Ginn. 8vo. Vol. 1, 1910, reprinted in 1916. 10 + 344 pages; price $4.00. Vol. 2, 1918. 12 + 511 pages; price, $5.00.

To dispel at the outset any misunderstanding concerning the following comment and criticism of the reviewer it must be stated that, in his opinion, Veblen and Young's Projective Geometry is a very scholarly and profound treatise on the axiomatic foundations of projective geometry, its classification, and the main body of projective propositions, or theorems. As such it undoubtedly reflects credit upon American scientific scholarship.

All through the two volumes one is constantly impressed with the fact that the axiomatic foundations are extremely important, and that the chain of principal theorems is merely a byproduct of axiomatic, and not the main object of, scientific research in general. This impression may of course be due to the fact that there are two irreconcilable classes of scientific minds: idealists and realists, just as in philosophy.

For this reason it is as a rule useless to argue a case belonging to the philosophy, or psychology, of this domain between representatives of the two different classes. The idealist maintains that the mind is free to create anything it pleases, without reference to any sense-perception whatsoever; mathematics in its foundation may be based upon pure logic alone. It is the purely logical process which, for him, is the essential thing. The realist, on the other hand, holds fast to the famous doctrine: "Nihil est in intellectu quod non prius fuerit in sensu." According to him our entire knowledge, fundamentally, may be traced back to the impressions made upon our mind by the influx through the senses. The choice